Alliance Consulting

BOND YIELDS & DURATION ANALYSIS

COMPUTING BOND YIELDS

Sources of returns on bond investments

The returns from investment in bonds come from the following:

1. Periodic coupon payments (except for zero coupon bonds)
2. Reinvestment income earned on the periodic coupon receipts
3. Capital gain or loss on sale of bond before maturity.

Illustration:

An investor who wishes to invest a certain amount of money for 5 years in the bond market has various choices available to him if the market is very liquid i.e. an active secondary market exists. The options include the following:

a. Invest in a bond with the **exact time to maturity of the investment horizon** i.e. 5 years. In this case, the investor will receive periodic interest payment i.e. coupon receipts and the principal on maturity.

   This investor's return is limited to sources 1 & 2 only above.

b. Invest in a **tenor shorter than that of the investment horizon** e.g. invest in a 2-year bond and then roll over for another 2 years by investing in another 2-year bond 2 years from now. On maturity of this bond 4 years from now, the investor can then invest in a 1-year bond. All previously received cash flows i.e. reinvestment income inclusive will be invested in each instance.

   In this case, this investor's return will consist of items 1 & 2 only also.

c. Invest in a **tenor longer than that of the investment horizon** e.g. invest in a 20-year bond and sell five years at the market price for 15-year bonds of similar quality.

   Under this scenario, the investor will receive income from all three sources outlined above.

MEASURING YIELD

The yield on any investment is the interest rate, y, that equalizes the present value of the expected cash flows from the investment to its price (cost) i.e. the interest rate, y, that satisfies the equation

$$P = \frac{CF_1}{1+y} + \frac{CF_2}{(1+y)^2} + \frac{CF_3}{(1+y)^3} + \ldots\ldots\ldots\ldots + \frac{CF_n}{(1+y)^n}$$

$$P = \sum \frac{CF_t}{(1+y)^t}$$

where: CF_t = cash flow in period t

P = price of the investment

n = number of periods

Note that for bonds, the last cash flow consists of periodic interest and the principal and is made up of the interest (C_i) and Principal repayment (P) i.e. CFn = C_i + P

Therefore for bonds the expression becomes:

$$P = \frac{C_1}{1+y} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + \ldots\ldots\ldots\ldots + \frac{C_i}{(1+y)^n} + \frac{P}{(1+y)^n}$$

$$P = \sum \frac{C_i}{(1+y)^n} + \frac{P}{(1+y)^n}$$

The yield calculated is also called the **Internal Rate of Return (IRR).**

Please note that the yield calculated is the yield for a period i.e. the period within which interest is paid.

Calculating the yield requires a trial and error approach (iterative process). However, for zero coupon bonds, there is no need to go through the iterative process as there are no intermediate cash flows hence determining yield is straight-forward.

$$P = \frac{CF_n}{(1 + y)^n}$$

Solving for yield, we have:

$$y = \left(\frac{CF_n}{P} \right)^{1/n} - 1$$

ANNUALIZING YIELDS

As indicated above, the yield calculated is the yield per period i.e. period for which interest is paid on the bond or investment. For comparative purposes, it is important to annualize the yield. To annualize, we calculate the Effective yield as follows:

$$\text{Effective Yield, } y_f = (1 + \text{Periodic interest rate, } y)^m - 1$$

Where m is the number of periods for which interest is paid in a year

CONVENTIONAL YIELD MEASURES

There are three measures of yield used in bond markets:

1. **Nominal Yield**
2. **Current Yield (CY)**
3. **Yield to Maturity (YTM)**
4. **Yield to Call (YTC)**
5. **Realized (Horizon) yield**

1. Nominal Yield

It is the coupon rate of a particular issue. A bond with an 8% coupon has an 8% nominal yield. It therefore provides a convenient way of describing the coupon characteristics of the bond.

2. Current Yield

Relates the annual coupon rate to market price i.e.

$$\text{Current Yield (CY)} = \frac{\text{Annual Dollar Coupon Interest}}{\text{Price}}$$

This yield measure relates the total nominal values of the annual coupon cash flows to the Market price. It therefore does not take the timing of these cash flows into consideration. Also, it does not take any other source of return on the bond into consideration e.g. capital gains or losses. It is however an important measure to income oriented investors.

3. Yield to Maturity

The Internal Rate of Return is the Yield to Maturity (YTM). Ideally, to annualize this return, we use the equation earlier discussed under Annualizing Yields. However in the bond market, the convention is to annualize by multiplying by a straight factor that converts the period to a year i.e. a factor of 2 for semi-annual bonds. When this is done, the annualized yield obtained is called the B***ond Equivalent Yield*. Please note that the Bond Equivalent Yield always understates the true or effective yield on a bond.**

The YTM calculation takes into account the time value of money by including the timing of the cash flows and the related capital gain or loss that the investor will realize by holding the bond to maturity. The relationship among the coupon rate, current yield and YTM is as follows:

Market Price of Bond	Expected relationship
Par	Coupon rate = Current yield = Yield to Maturity
Discount	Coupon rate < Current yield < Yield to Maturity
Premium	Coupon rate > Current yield > Yield to Maturity

The YTM formula assumes that the bond is held to maturity and all interim cash flows were reinvested at the YTM rate because it discounts all cash flows at that rate.

The impact of the reinvestment assumption on the actual return of a bond varies directly with the bond's coupon and term to maturity. A higher coupon rate and term to maturity will increase the loss in value from failure to reinvest at the YTM. Therefore, a higher coupon or longer maturity makes the reinvestment assumption more important.

4. Yield to Call (YTC)

For a bond that may be called prior to its stated maturity date, another yield measure is commonly quoted – ***the Yield to Call (YTC).*** The method of calculating YTC is the same as for calculating the YTM except that the cash flows used in the computation are those that are expected to occur before the first call date.

YTC is therefore the yield that will make the present value of the cash flows up to first call date equal to the price of the bond i.e. assuming the bond is held to its first call date.

$$P = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \frac{C}{(1+y)^3} + \ldots\ldots\ldots\ldots + \frac{C}{(1+y)^{n*}} + \frac{M*}{(1+y)^{n*}}$$

$M*$ = call price (in dollars)

$n*$ = number of periods until first call date

Investors normally calculate the YTC and YTM and select the lower of the two as a measure of return.

Again, like the YTM measure the YTC measure also assumes the following:

- Reinvestment of interim cash flows at the YTC rate
- The bond is held to its first call date

5. Horizon Yield

This measures the expected rate of return of a bond that you expect to sell prior to its maturity. It is therefore a total return measure which, allows the portfolio manager to project the performance of a bond on the basis of a planned investment horizon, his expectations concerning reinvestment rates and future market yields. This allows the portfolio manager to evaluate which of several potential bonds considered for investment will perform best over the planned investment horizon.

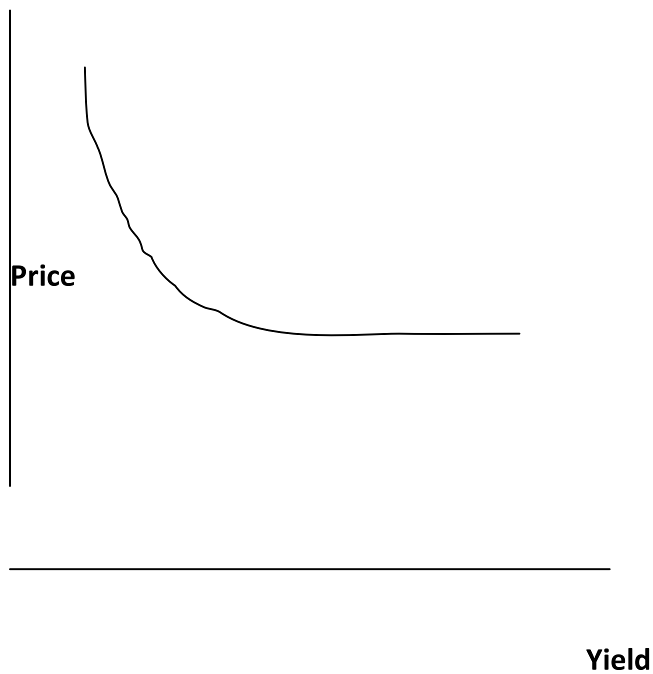
Using total return to assess performance over some investment horizon is called ***Horizon Analysis*** while the return calculated over the horizon is called ***Horizon Yield or Return.***

The disadvantage of this approach for calculating return is that it requires the portfolio manager to make some assumptions about reinvestment rates, future yields and to think in terms of a specified period or horizon. It however enables the manager to evaluate the performance of a bond under different interest rate scenarios thereby assessing the sensitivity of the bond to interest rate changes.

BOND PRICE VOLATILITY

Recall the relationship between bond prices and changes in interest rate or yield for an option free bond.

An increase (decrease) in the required yield on a bond will result in a decrease (increase) in the bond's price. When the price-yield relationship is graphed, it exhibits the shape shown below:



Note that the relationship is not linear, the shape of the price-yield relationship is referred to as convex.

Bond price volatility is measured in terms of percentage change in price. Bond price volatility is influenced by more than yield behavior alone. Factors that affect it are:

- Par value
- Coupon rate
- Term to maturity
- Prevailing interest rate

The following relationship exists between yield and bond price behavior:

- Prices move inversely to yield
- For a given change in yield, longer maturity bonds post larger price changes thus bond price volatility is directly related to term to maturity.
- Price volatility increases at a diminishing rate as term to maturity increases
- Price movements resulting from equal absolute increases or decreases in yield are not symmetrical. A decrease in yield raises bond prices by more than an increase in yield of the same amount lowers prices.
- Higher coupon issues show smaller percentage fluctuations for a given change in yield i.e. price volatility is inversely related to coupon.

ILLUSTRATION 1:

Effect of Maturity on Bond Price Volatility

---------------- Present Value of an 8% Bond ($1,000 Par Value) ------

	1 Year		10 Years		20 Years		30 Years	
Yield to Maturity	**7%**	**10%**	**7%**	**10%**	**7%**	**10%**	**7%**	**10%**
Present value: **- interest**	75	73	569	498	858	686	1,005	757
- principal	934	907	505	377	257	142	132	54
Total value	**1,009**	**980**	**1,074**	**875**	**1,115**	**828**	**1,137**	**811**
% change		**-2.9%**		**-18.5**		**-25.7%**		**-28.7%**

ILLUSTRATION 2:

Effect of Coupon on Bond Price Volatility

-----------Present Value of an 20 Year Bond ($1,000 Par Value) ------

	0 Coupon		**3% Coupon**		**8% Coupon**		**12% Coupon**	
Yield to Maturity	**7%**	**10%**	**7%**	**10%**	**7%**	**10%**	**7%**	**10%**
Present value: **- interest**	0	0	322	257	858	686	1,287	1,030
- principal	257	142	257	142	257	142	257	142
Total value	**257**	**142**	**579**	**399**	**1,115**	**828**	**1,544**	**1,172**
% change		**-44.7%**		**-31.1**		**-25.7%**		**-24.1%**

MEASURES OF BOND PRICE VOLATILITY

A measure of interest rate sensitivity of a bond is called DURATION. There are three key measures and they are:

1. Macaulay duration
2. Modified duration
3. Effective duration

MACAULAY DURATION is a measure of the time flow from a bond. It can also be likened to the weighted average number of years over which a security's total cash flows occur. The weightings used are the market value of the cash flows.

$\Sigma \, \underline{C_t \, (t)}$

$$D = \frac{\sum \frac{C_t}{(1+i)^t} \cdot (1+i)^t}{\text{Price of the bond}}$$

It therefore seeks to measure the time characteristics of the bond.

EXAMPLE:

Consider two bonds with the following features:

Face value	$1,000	$1,000
Maturity	10 years	10 years
Coupon	4%	8%

Calculate the Macaulay duration for each of the bonds assuming an 8% market yield.

SOLUTION:

Bond A

Year	**Cash flow**	**PV @ 8%**	**PV of flow**	**PV as % of Price**	**PV as % of Price time weighted**
1	40	.9259	37.04	.0506	.0506
2	40	.8573	34.29	.0469	.0938
3	40	.7938	31.75	.0434	.1302
4	40	.7350	29.40	.0402	.1608
5	40	.6806	27.22	.0372	.1860
6	40	.6302	25.21	.0345	.2070
7	40	.5835	23.34	.0319	.2233
8	40	.5403	21.61	.0295	.2360
9	40	.5002	20.01	.0274	.2466
10	1,040	.4632	481.73	.6585	6.5850
Total			731.58	1.0000	8.1193

Duration = 8.12 years

Bond B

Year	Cash flow	PV @ 8%	PV of flow	PV as % of Price	PV as % of Price time weighted
1	80	.9259	74.07	.0741	.0741
2	80	.8573	68.59	.0686	.1372
3	80	.7938	63.50	.0635	.1906
4	80	.7350	58.80	.0588	.1906
5	80	.6806	54.44	.0544	.2720
6	80	.6302	50.42	.0504	.3024
7	80	.5835	46.68	.0467	.3269
8	80	.5403	43.22	.0432	.3456
9	80	.5002	40.02	.0400	.3600
10	1,080	.4632	500.26	.5003	5.0030
Total			1,000.00	1.0000	7.2470

Duration = 7.25 years

Characteristics

1. Macaulay duration of a bond with coupon payment will be less than its term to maturity because of the interim cash flows.
2. There is an inverse relationship between coupon and duration i.e. the higher the coupon the lower the duration.
3. There is a positive relationship between term to maturity and Macaulay duration but duration increases at a decreasing rate with maturity.
4. There is an inverse relationship between YTM and duration, all other things being equal.
5. Sinking funds and call provisions have dramatic effect on a bond’s duration.

MODIFIED DURATION – measures the price volatility of a non-callable bond.

$$\text{Modified Duration} = \frac{\text{Macaulay Duration}}{1 + YTM/n}$$

Where :- YTM - the yield to maturity of the bond

n - number of payments per year

Please note that the greater the modified duration the greater the price volatility of the bond for small changes in yields. Specifically, an estimate of the percentage change in price equals the change in yield times the modified duration.

$$\text{Therefore } \frac{dP}{dY}\frac{1}{P} = -\text{modified duration}$$

EXAMPLE:

Consider a bond with Macaulay duration of 8 years, yield (i) of 10%. Assume you expect the YTM to decline by 75 basis point (say from 10% to 9.25%). The modified duration is as follows.

$$\text{Modified Duration} = \frac{8 \text{ years}}{(1 + 0.1/2)} = 7.62 \text{ years}$$

the estimated percentage price change in the price of the bond is

$$\%\text{age change in P} = -7.62 \times \frac{-75}{100}$$

$$= 5.72$$

Note: The modified duration is always a negative for non-callable bonds because of the inverse relationship between price and yield. Also, modified duration provides a good estimate of price change for only small changes in yield of option free securities.

If you expect a decline in interest rates, you should increase the average modified duration of your bond portfolio to experience maximum price volatility.

Duration changes in a non-linear fashion with yield changes – a concept called convexity. It therefore requires the recalculation and rebalancing as rate changes.

EFFECTIVE DURATION is a direct measure of the interest rate sensitivity of a bond or any asset where it is possible to observe the market prices surrounding a change in interests.

%age change in Price = - Modified Duration X change in Yield

$$-D^* = \frac{\text{\%age change in Price}}{\text{Change in yield}}$$

The D* obtained this way is the effective duration.

Example:

Interest rate decline is 200 basis point

Price of the bond increases by 10%

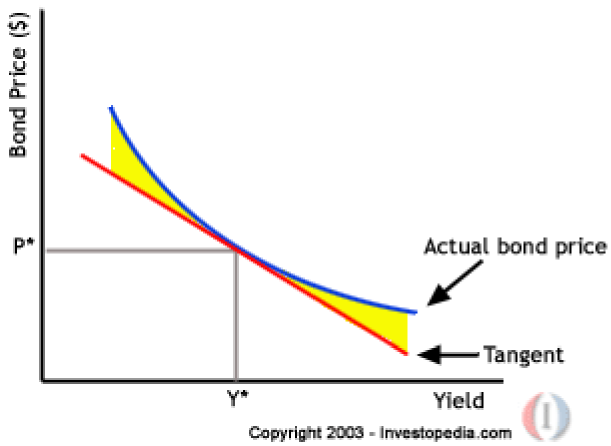
$$\text{Then} \quad \text{Effective Duration} = \frac{10}{-200/100} = 5$$

Note:

1. It is possible to have an Effective duration greater than maturity as in the case of CMO.
2. It is also possible to compute a negative effective duration as in the case of bonds with embedded options.e.g. mortgage-backed securities.

CONVEXITY

For any given bond, a graph of the relationship between price and yield is convex. This means that the graph forms a curve rather than a straight-line (linear). The degree to which the graph is curved shows how much a bond's yield changes in response to a change in price.

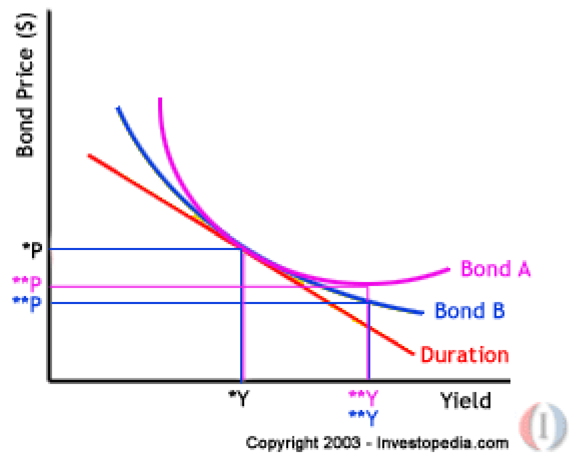


Furthermore, as yield moves further from Y*, the yellow space between the actual bond price and the prices estimated by duration (tangent line) increases.

The convexity calculation, therefore, accounts for the inaccuracies of the linear duration line. This calculation that plots the curved line uses a Taylor series, a very complicated calculus theory that we won't be describing here. The main thing for you to remember about convexity is that it shows how much a bond's yield changes in response to changes in price.

PROPERTIES OF CONVEXITY

Convexity is also useful for comparing bonds. If two bonds offer the same duration and yield but one exhibits greater convexity, changes in interest rates will affect each bond differently. A bond with greater convexity is less affected by interest rates than a bond with less convexity. Also, bonds with greater convexity will have a higher price than bonds with a lower convexity, regardless of whether interest rates rise or fall. This relationship is illustrated in the following diagram:

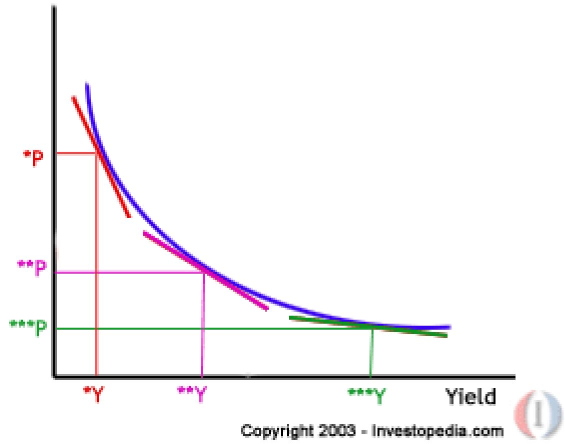


As you can see Bond A has greater convexity than Bond B, but they both have the same price and convexity when price equals *P and yield equals *Y. If interest rates change from this point by a very small amount, then both bonds would have approximately the same price, regardless of the convexity. When yield increases by a large amount, however, the prices of both Bond A and Bond B decrease, but Bond B's price decreases more than Bond A's. Notice how at **Y the price of Bond A remains higher, demonstrating that investors will have to pay more money (accept a lower yield to maturity) for a bond with greater convexity.

What Factors Affect Convexity?

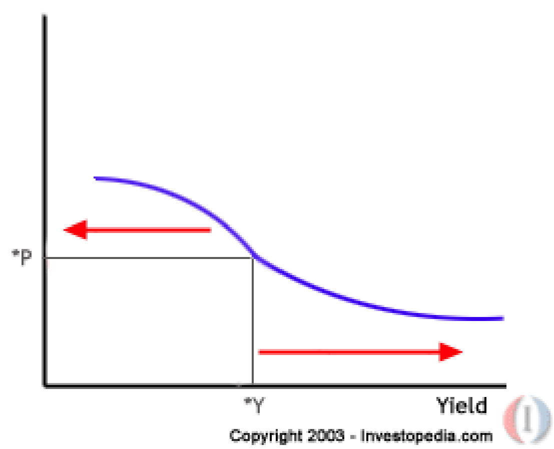
Here is a summary of the different kinds of convexities produced by different types of bonds:

1) The graph of the price-yield relationship for a plain vanilla bond exhibits positive convexity. The price-yield curve will increase as yield decreases, and vice versa. Therefore, as market yields decrease, the duration increases (and vice versa).



2) In general, the higher the coupon rate, the lower the convexity of a bond. Zero-coupon bonds have the highest convexity.

3) Callable bonds will exhibit negative convexity at certain price-yield combinations. Negative convexity means that as market yields decrease, duration decreases as well. See the chart below for an example of a convexity diagram of callable bonds.



Remember that for callable bonds, modified duration can be used for an accurate estimate of bond price when there is no chance that the bond will be called. In the chart above, the callable bond will behave like an option-free bond at any point to the right of *Y. This portion of the graph has positive convexity because, at yields greater than *Y, a company would not call its bond issue: doing so would mean the company would have to reissue new bonds at a higher interest rate. Remember that as bond yields increase, bond prices are decreasing and thus interest rates are increasing. A bond issuer would find it most optimal, or cost-effective, to call the bond when prevailing interest rates have declined below the callable bond's interest (coupon) rate. For decreases in yields below *Y, the graph has negative convexity, as there is a higher risk that the bond issuer will call the bond. As such, at yields below *Y, the price of a callable bond won't rise as much as the price of a plain vanilla bond.

Convexity is the final major concept you need to know for gaining insight into the more technical aspects of the bond market. Understanding even the most basic characteristics of convexity allows you to better comprehend the way in which duration is best measured and how changes in interest rates affect the prices of both plain vanilla and callable bonds.